applied to the combination limiter. Fig. 4 shows a photograph of the oscilloscope presentation of a 1N23 detector on the output of the second-stage limiter. The input pulse, attenuated 53 db, is shown for comparison. Measurements indicate that the total spike leakage is 0.2 erg of energy. The maximum power available for this particular test was 25 kw. There were no indications of either diode or limiter failure, and it could be reasonably expected that the combination could withstand higher power.

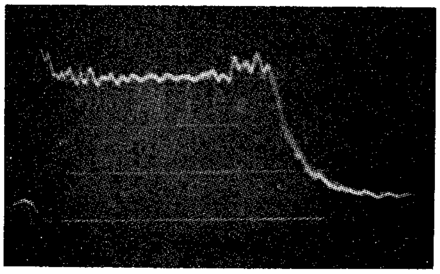

Fig. 2—Direct output of the gyromagnetic coupling limiter through a 1N23 detector. Time base is 0.2 μsec per square.

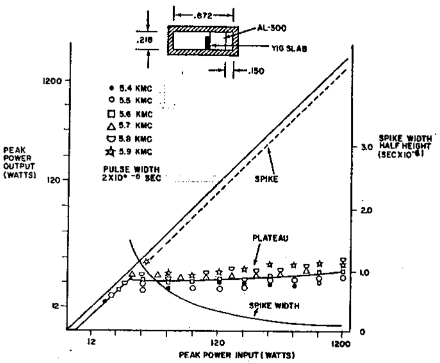

Fig. 3—Characteristics of the subsidiary resonance first-stage limiter.

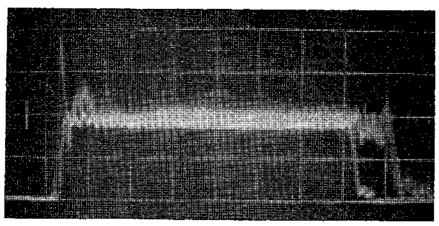

Fig. 4—Direct output of the two-stage limiter through a 1N23 detector. The input attenuated 53 db is shown for comparison. Time base is 0.2 μsec per square.

In conclusion, it has been shown that receiver protection can be afforded by ferrimagnetic limiters. These limiters are passive, and when used as specified should have no apparent lifetime limitations.

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Temporal Instabilities in Traveling-Wave Parametric Amplifiers*

The basic traveling-wave parametric amplifier (TWPA) [1]–[3], as here defined, consists of an all-pass uniform transmission-line structure in which the distributed circuit elements are modulated in time and space by a progressive pumping wave. TWPA's, in general, have aroused great interest due to the possibilities of wide-band amplification, as predicted by coupled mode theory [2]–[4]. It is the purpose of this communication to show that temporal instabilities exists on the basic TWPA (or its dual [1]) when the frequency relations are of the negative-resistance type (the inverting modulator of Manley and Rowe [6]). It is found, from the exact solution for time harmonic waves existing on this line, that under these conditions waves growing in time are present, rather than waves growing in distance along the line.

The exact solution for time harmonic waves on the uniform transmission line, which possesses distributed coefficients periodically modulated in time and space (Fig. 1), is [7]

$$V(z, t) = e^{j(\omega t - kz)} \sum_{n=-\infty}^{\infty} V_n e^{j(\omega_1 t - k_1 z)n} \quad (1)$$

where

ω = angular frequency of the "signal"
k = propagation wavenumber of the "signal"
ω_1 = angular frequency of pumping wave (specified)
k_1 = propagation wavenumber of pumping wave
$= \omega_1/c'$
c' = phase velocity of pumping wave (specified).

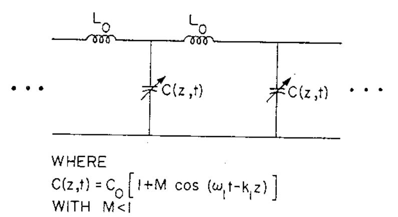

Fig. 1—Basic traveling-wave parametric amplifier.

It may be shown that this solution is a mathematically complete solution and is valid everywhere except [7], [8] where the pump phase velocity (c') is within a specified range near the propagation velocity of the line [$v_{p0} = (L_0C_0)^{-1/2}$]. This range has been called the "sonic region" [7], [8], and its width is dependent on the amplitude of modulation, M [7], [8].

The differential equation governing the modulated transmission line is as follows:

$$\frac{\partial^2 V(z, t)}{\partial z^2} = L_0 \frac{\partial^2}{\partial t^2} [C(z, t)V(z, t)]. \quad (2)$$

The substitution of (1) into (2) leads to the following three-term recursion relation [7], [8], [10], [11] for the time-space harmonic amplitudes

$$V_{n+1} + D_n V_n + V_{n-1} = 0, \quad (3)$$

where

$$D_n = \frac{2}{M}\left[1 - \left(\frac{ka + 2\pi n}{k_{n_0}a}\right)^2\right]$$

$$k_{n_0}a = k_0 a + 2\pi mn, \quad k_0 = \omega/v_{p_0}, \quad a = 2\pi/k_1$$

$$v_{p_0} = (L_0C_0)^{-1/2}, \quad m = c'/v_{p_0}.$$

Furthermore, it has been shown [9] that such a recursion relation may be used to derive a dispersion relation (between ω and k) in the form of infinite continued fractions in the D_n's as follows:

$$D_n - \cfrac{1}{D_{n-1} - \cfrac{1}{D_{n-2} - \cfrac{1}{\ddots}}} - \cfrac{1}{D_{n+1} - \cfrac{1}{D_{n+2} - \cfrac{1}{\ddots}}} = 0. \quad (4)$$

Solutions of (4) determine the relationship of $k(\omega)$ or $\omega(k)$, for specified parameters ω_1, c', and M, to any degree of accuracy desired. Previous analyses of this type [10], [11] have not considered interaction effects of all harmonics (n) in obtaining the propagation characteristics.

The dispersion relation has been investigated in two cases herein, outside of the sonic region, where the solutions are valid. The first case examined is that for which the pump velocity is less than the propagation velocity ($c' < v_{p0}$). Here, all interactions take place where the frequency condition $\omega > \omega_1$ is satisfied. The results are shown in Fig. 2, for the fixed parameters $M = 0.25$ and $m = c'/v_{p0} = 0.25$, with the plot normalized to a third parameter: $a = 2\pi/k_1$. The dispersion curve is that typical of a "stop-band" in spatially periodic structures and, indeed, the static modulation case occurs here in the limit $c' = 0$.

The region of the Brillouin diagram shown in Fig. 2 is that of the principal interaction of the fundamental wave ($n = 0$, forward group velocity) with the first harmonic ($n = -1$, backward group velocity). In the absence of interaction ($M = 0$), the two waves would be represented on the Brillouin diagram by two straight lines at $\pm 45°$ (not shown) intersecting in the middle of the stop-band. The corresponding lines for any of the harmonics for this structure are described in general by

$$ka = -2\pi n \pm (k_0 a + 2\pi mn). \quad (5)$$

* Received by the PGMTT, September 20, 1961. This work was sponsored by the A F Cambridge Res. Labs. under Contract AF 19(604)-7499.

The points of intersection of such lines represents the regions of strong interaction between harmonics. It may be recognized that such intersections for the present basic structure occur only between waves of opposite group velocities, since waves with group velocities in the same direction are represented by parallel lines.

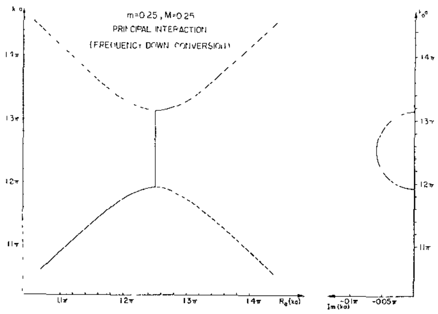

Fig. 2—Brillouin diagram—basic TWPA—noninverting modulator case.

The stop-band region shown in Fig. 2 reveals complex values of propagation constants (k), indicating spatially growing and decaying waves for this interaction, and corresponds to parametric down-conversion of frequencies if ω is considered the input signal. The result in this case is qualitatively the same as that found from coupled-mode theory [2], [4], [5] insofar as exponentially varying waves are found when signal and idler waves have oppositely directed group velocities (*i.e.*, contradirectional coupling [4]). Finally we note that the real part of the propagation constant (k) is unvarying throughout the stop-band—a well-known phenomenon in purely spatially-modulated structures.

The second and more interesting case treated here is that where the pump velocity is greater than the propagation velocity of the unmodulated transmission line. In this case the interactions take place only in regions where $\omega < \omega_1$, thus corresponding to conditions of the negative-resistance type of parametric amplifier (the inverting modulator [6]). In Fig. 3 results are shown for the evaluation of the dispersion relation for the case $m = 1.5$ and $M = 0.25$. Again, we are considering the region for which the dominant interaction occurs between the forward-traveling fundamental ($n = 0$) and the first backward-traveling harmonic ($n = -1$). Here we note a striking behavior for the dispersion curve, which shows a *stop-band in the temporal frequency* ($k_0 = \omega/v_{p0}$).

A dispersion curve of the type shown in Fig. 3 has been discussed by Sturrock [13], who refers to this type of behavior as a "non-convective instability." Sturrock's discussion is general, covering all types of waves, but his applications of the theory to date have been to approximate dispersion relations for electron devices [13] and plasmas [14]. The case considered herein is, by contrast, an exact dispersion relation and for purely electromagnetic waves, wherein we recognize regions of solutions to the dispersion relation which are of the transient type (*i.e.*, a complex temporal frequency).

Simon [10] has stated that the conditions described here ($\omega < \omega_1$) are similar to those of the "carcinotron" and applies his approximate analysis to predict oscillations. The oscillations are predicted by Simon on the basis of approximate solutions for the fundamental and idler waves in a pumped section of medium of finite length. He finds that the two waves must start from zero values inside of the section, each from opposite ends of the section. This, in turn, may be reconciled outside of the pumped region only by allowing the two waves to propagate away from the section in opposite directions from one another. Simon concludes that such conditions are self-oscillatory. The dispersion relation method described in the present note, however, predicts the existence of temporally growing harmonic solutions for the electromagnetic waves in a progressively pumped medium of infinite length. Furthermore, the frequency of oscillation of this "backward-wave parametric oscillator" may be predicted exactly from the real part of k_0, which remains constant throughout the inverted stop-band.

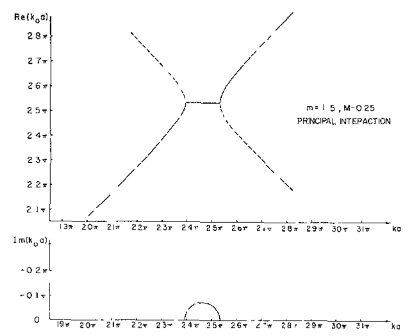

Fig. 3—Brillouin diagram—basic TWPA—inverting modulator case.

Eidmon [15] has recognized the two frequency conditions described above as being distinct from one another in the case of interacting plasma waves. Eidmon calls the harmonics "anomalous Doppler frequencies" for the case $\omega < \omega_1$, in contrast to "normal Doppler frequencies" for the case $\omega > \omega_1$.

In conclusion, we should note that not only does the exact solution given here predict temporal instabilities for the basic TWPA when $\omega < \omega_1$, but, furthermore, no solutions of the spatially growing type exist for this elementary structure. This last-mentioned comment is based on the absence of interactions between waves having group velocities of the same sign for the simple structure possessing distributed constants. Sturrock [13] states that true amplification is possible only when the group velocities of the two interacting waves are in the same direction (*i.e.*, codirectional coupling [4]). It is believed, therefore, that (spatial) amplification will occur on parametric traveling-wave circuits, as originally predicted by the Tien [12] theory, but only on structures capable of supporting waves having group velocities of like signs, which allow interactions for the case $\omega < \omega_1$. This last point has been demonstrated explicitly for the case of an iterated chain of waveguide cavities by Currie and Gould [16], using a method of analysis similar to that used here. Their results are verified experimentally by Grabowski and Weglein [17]. It is conjectured that the same situation exists as an interpretation of other experimental results [3], [18], [19] on traveling-wave parametric amplification. On the other hand, it is also possible that temporal (nonconvective [13]) instabilities exist on any of these structures. The existence of such instabilities may be predicted, as stated above, by ascertaining if intersections of the unpumped dispersion loci occur for waves of opposite group velocities (contradirectional waves) when the frequency condition $\omega < \omega_1$ is satisfied.

Further results and interpretation on this subject are in progress by the writer and will be reported at a later date. The assistance of J. Siegel is gratefully acknowledged for his programming of the solution of the dispersion relation on the IBM 650 computer.

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